Rolling Carts: Conservation of Energy with a Spring and Ramp

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Section B
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INTRODUCTION

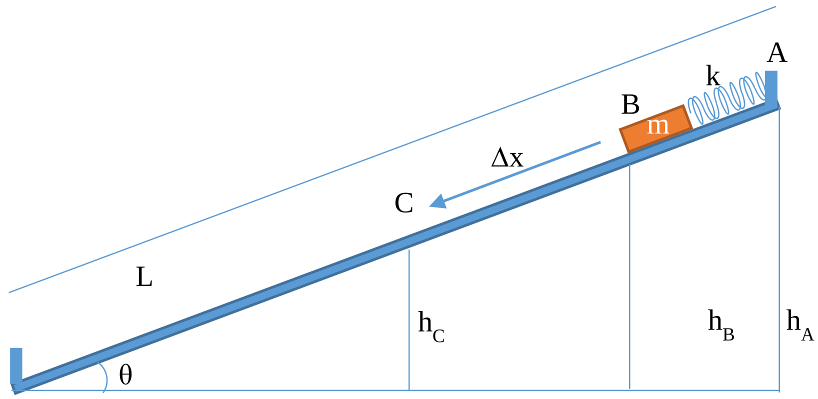
Purpose: The purpose of the lab was to design and perform an experiment which analyzes the conservation of energy in a spring-based system.

Researchable Question: How does increasing the height of one end of a ramp affect the distance a cart travels when it is attached to the top of the ramp with a spring?

Hypothesis: If the height of one end of a ramp is increased, then the distance that a cart, fixed to the top of the ramp with a spring, travels will increase, where height is directly proportional to the distance.

METHODOLOGY

Diagram:



Procedure: One end of the ramp was secured to the floor using duct tape, forming a makeshift hinge. A chair was used to prop up the other end of the ramp, which was placed at various heights. All measurements were done from the bottom edge of the ramp. Changing the height directly changed the angle of the ramp in an easier and more effective manner. A spring was attached to the cart through a small preexisting hole. The spring was then attached to the top of the ramp and secured firmly with a piece of tape. The cart was lowered to 2.0 meters from the bottom of the ramp, where the spring would not be under any tension nor slack. Two observers were positioned above the ramp, and the cart was released. The distance that the cart traveled was recorded in an Excel spreadsheet. These tests were repeated ten times at each of the six height settings.

Materials:

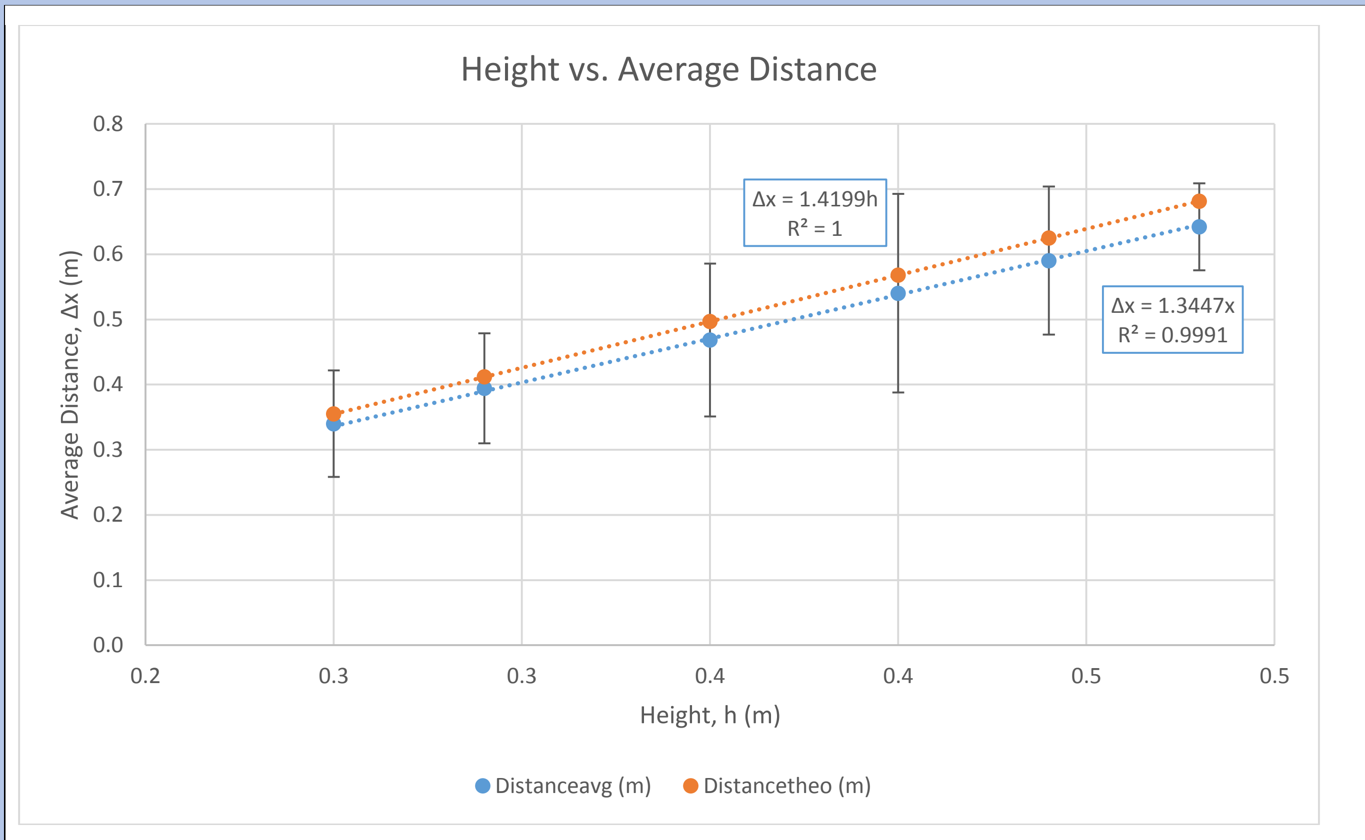
- Extruded Aluminum Ramp
- Low Friction Cart
- Meter Stick; Force Scale
- Spring

CONSTANTS/EQUATIONS

Constants:	**Equations:**
$m = 502.1\ g = 0.5021\ kg$	Conservation of Energy: $\sum E_i = \sum E_f$
$L = 228.5\ cm = 2.285\ m$	$\sum E_i = PE_{gB}; \sum E_f = PE_{gC} + PE_{sC}$
$g = 9.8\ m/s^2$	$PE_{gB} = mgh_B; PE_{gC} = mgh_C;\ PE_s = \frac{1}{2}k(\Delta x)^2$
	$h_B = 2 * \sin(\theta)\,; h_C = (2 - \Delta x) * \sin(\theta)$
$k = 3.033\ N/m$ (See Appendix B)	$\theta = \sin^{-1}\frac{h_A}{L}; \Delta x = \frac{2 * \mathrm{mg} * \sin(\theta)}{k}$ (See Appendix C)

SUMMARIZED DATA

	Height (m)	θ (rad)	$Distance_{avg}$ (m)	STDEV (m)	%RSD of D_{avg}	$Distance_{theo}$ (m)	%ERR (m)	Σe_f (avg) (J)	Σe_f (theo) (J)	% Change of Energy
1	0.480	0.212	0.6420	0.001	0.10%	0.6815	5.80%	2.029	2.067	1.86%
2	0.440	0.194	0.5902	0.001	0.19%	0.6247	5.53%	1.864	1.895	1.63%
3	0.400	0.176	0.5401	0.002	0.28%	0.5679	4.90%	1.700	1.723	1.32%
4	0.290	0.127	0.3944	0.001	0.21%	0.4118	4.22%	1.239	1.249	0.83%
5	0.350	0.154	0.4684	0.001	0.25%	0.4970	5.75%	1.487	1.507	1.35%
6	0.250	0.110	0.3400	0.001	0.24%	0.3550	4.22%	1.069	1.077	0.72%
			Average:	**0.001**	**0.21%**	**Average:**	**5.07%**		**Avg:**	**1.29%**



CONCLUSION

As the angle of the ramp is increased by increasing the height of one end of the ramp, the cart travels further down the ramp. Although the data supports our hypothesis, the conservation of energy is not demonstrated in this experiment, most evident by the force of friction. In addition, as the spring was continuously flexed, a significant amount of energy was converted into thermal energy, resulting in additional loss of energy.

ANALYSIS

The average |%RSD| of the data is 0.21%, indicating high precision. The average |%ERR| is 5.07%, which represents high accuracy in the data. The mathematical model for the both the measured and theoretical data sets are strong, as they are 0.9991 and 1, respectively. The mathematical model only applies to ramp angles between 0° and 90°. At angles outside of this range, the cart would not be able to roll down the ramp and stretch the spring. The percentage of the energy change is consistently low at an average loss of 1.29%. The major source of error is in human judgement, as the cart rolls to a stop and begins moving too quickly for the human eye to measure exactly. If high speed video was used, a better measurement could be obtained. The statistical analysis of the data, including standard deviation, percent RSD, and percent energy loss indicate a high precision experiment with minimal errors. However, friction was not factored in to any calculations, introducing a possible source of error. The data proves the hypothesis that as the angle of the ramp is increased, the distance that the cart (attached to the top of the ramp by a spring) travels also increases in direct proportionality.

PHOTOGRAPHS

APPENDIX A

	Height (cm)	Trial 1 (cm)	Trial 2 (cm)	Trial 3 (cm)	Trial 4 (cm)	Trial 5 (cm)	Trial 6 (cm)	Trial 7 (cm)	Trial 8 (cm)	Trial 9 (cm)	Trial 10 (cm)	Obtained D_{avg} (cm)	STDEV (cm)
IV 1	48.0	135.9	135.8	135.8	135.7	135.8	135.9	135.8	135.8	135.7	135.8	135.8	0.0667
IV 2	44.0	141.1	140.8	141.0	141.1	141.0	140.8	141.1	140.9	141.0	141.0	141.0	0.1135
IV 3	40.0	146.3	146.1	145.9	146.0	146.1	146.0	145.8	145.8	146.0	145.9	146.0	0.1524
IV 4	29.0	160.5	160.6	160.6	160.5	160.5	160.4	160.6	160.6	160.6	160.7	160.6	0.0843
IV 5	35.0	153.2	153.0	153.1	153.2	153.3	153.1	153.3	153.1	153.0	153.3	153.2	0.1174
IV 6	25.0	166.0	166.1	166.1	166.0	165.9	166.0	166.1	165.9	165.9	166.0	166.0	0.0816

Height (m)	θ (rad)	$Distance_{avg}$ (m)	STDEV (m)	|%RSD| of D_{avg}	$Distance_{theo}$ (m)	|%ERR| (m)	Σe_i (J)	Σe_f (avg) (J)	Σe_f (theo) (J)	% Change of Energy
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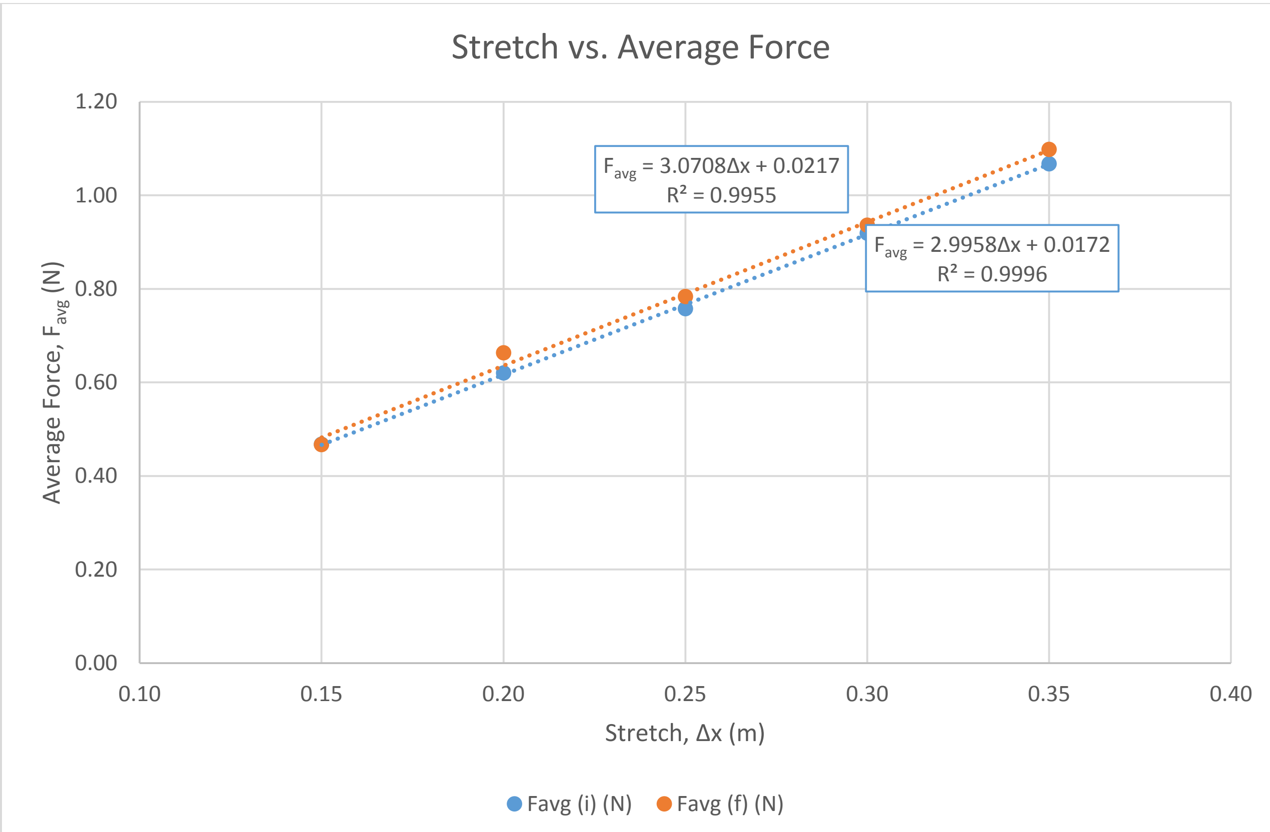
APPENDIX B

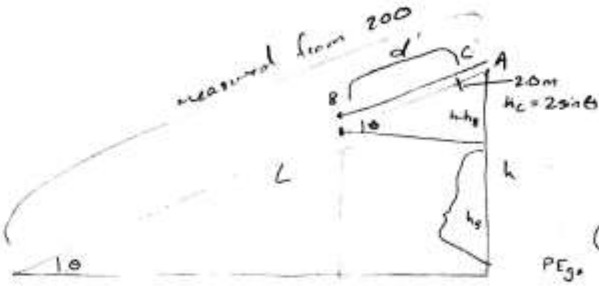


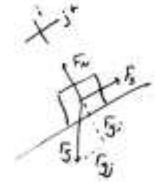
Stretch (cm)	Stretch (m)	F_{avg} (i) (N)	F_{avg} (f) (N)
15.0	0.150	0.4677	0.4668
20.0	0.200	0.6197	0.6629
25.0	0.250	0.7576	0.7832
30.0	0.300	0.9190	0.9359
35.0	0.350	1.0670	1.0980

$k_{initial}$ (N/m)
2.996

k_{final} (N/m)
3.071

APPENDIX C





(PE_gc) substitute for PE_A + PE_s

$PE_{gA} + KE_A^0 + PE_s = PE_{gB} + KE_B^0$

$PE_{gA} - PE_s = PE_{gB}$

$mgh_A - \frac{1}{2}k\Delta x^2 = mgh_B$

$mgh - \frac{1}{2}kd^2 = mg(h - d\sin\theta)$

$d = \frac{2mg\sin\theta}{k}$

k = 3.0733

mass = 0.5021 kg

L = 0.2285 m

$\theta = \sin^{-1}(h/L)$

$h_B = h - d\sin\theta$

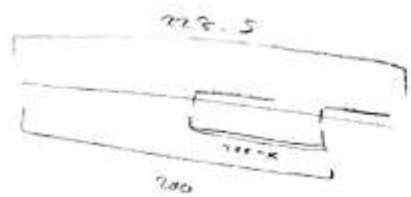
$\sin\theta = \frac{h}{L}$

$\theta = \sin^{-1}\left(\frac{h}{L}\right)$

$\sin\theta = \frac{h - h_B}{d}$

$d\sin\theta = h - h_B$

$h - d\sin\theta = h_B$



$Energy_{lost} = E_f - E_i$

$PE_{gc} - (PE_{gB} + PE_{sB})$

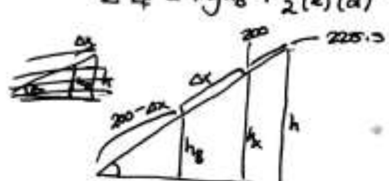
mgh_c

$h_c =$

$PE_{gB} = mgh_B$ $\quad \Sigma E_f = PE_{gB} + PE_{sB}$

$PE_{sB} = \frac{1}{2}k(\Delta x)^2$

$\Sigma E_f = mgh_B + \frac{1}{2}(k)(d)^2$



$\sin\theta = h_x/200$ $\quad \sin\theta = h_B/(200-\Delta x)$

$200\sin\theta = h_x$ $\quad (200-\Delta x)\sin\theta = h_B$